The Discontinuity Problem of a Rectangular Dielectric Post in a Rectangular Waveguide

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Abstract—**A simple numerical technique for the solution of the discontinuity problem of a symmetric loaded rectangular dielectric post centered in a rectangular waveguide is presented. The waveguide is divided into three regions where the field is expressed in suitable waveguide modes. By applying the continuity conditions at the common surfaces of the regions, a system of linear equations determining the reflection and transmission coefficients is formed. Several examples are compared with experimental results and show the validity of the method.**

I. Introduction

Various methods for studying the post interaction in the microwave region have been proposed [1]–[13]. A post offers the advantage of several applications by using a small amount of material. So instead of filling the entire cross section of a waveguide for permittivity and conductivity measurements or for filter design we can use posts.

A number of comprehensive works have dealt with circular posts by using approximate analytical or numerical methods [1]–[12]. In this paper we study posts of rectangular cross section which can be made easily. An excellent study of such a problem has been made recently by Yoshikado and Taniguchi [13]. In their work, an approximate analytical solution for the Helmholtz equation was derived and a simple method for measuring the complex conductivity of a square lossy dielectric post was established. The analysis of Yoshikado and Taniguchi has shown that there are cases for which there is no agreement between the theoretical and experimental results. This is due to their formulation, which does not satisfy the boundary conditions at the four corners of the post. In our work we give the field expression in the interaction region as a function of the partially filled rectangular waveguide modes. We thus give a better physical description of the field and the results become more reasonable.

II. Formulation

We start from a short presentation of a strip-loaded rectangular waveguide. The propagation problem of this configuration has been treated extensively by Lewin [14].

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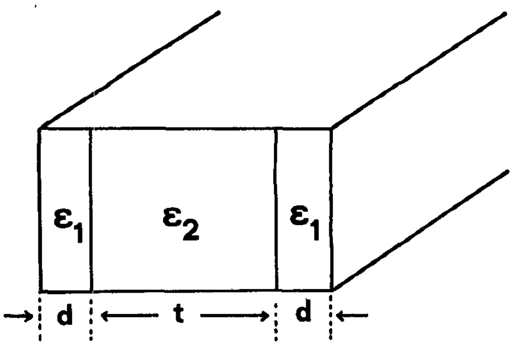

Fig. 1. Geometry of a symmetric strip loaded waveguide.

Using his formulation for a symmetric geometry (see Fig. 1), we can find the eigenvalue equation which gives the propagation constant [13]:

$$\tan(h_2 t) + 2\frac{h_2}{h_1}\frac{\tan(h_1 d)}{1-\left(\frac{h_2}{h_1}\right)^2 \tan^2(h_1 d)} = 0. \tag{1}$$

Equation (1) can be split into two equations:

$$\frac{h_2}{h_1}\tan(h_1 d) - \cot\left(\frac{h_2 t}{2}\right) = 0$$
$$\frac{h_2}{h_1}\tan(h_1 d) + \tan\left(\frac{h_2 t}{2}\right) = 0 \tag{2}$$

where h_1 and h_2 are given by

$$h_1^2 = k_{\mathrm{I}}^2 - k_1^2$$
$$h_2^2 = k_{\mathrm{II}}^2 - k_1^2 \tag{3}$$

where $k_{\mathrm{I}}^2 = \omega^2\mu\epsilon_1$, $k_{\mathrm{II}}^2 = \omega^2\mu\epsilon_2$, and k_1 is the unknown transmission coefficient of the waveguide.

For a given frequency, (1) or (2) can have more than one solution. Each solution gives the corresponding mode of the waveguide. For a real value of k_1, we have a transmitting mode while for an imaginary value we have an evanescent mode.

Let us now look at the geometry of a rectangular waveguide which is loaded with a rectangular post (Fig. 2). The space in the waveguide is divided into three

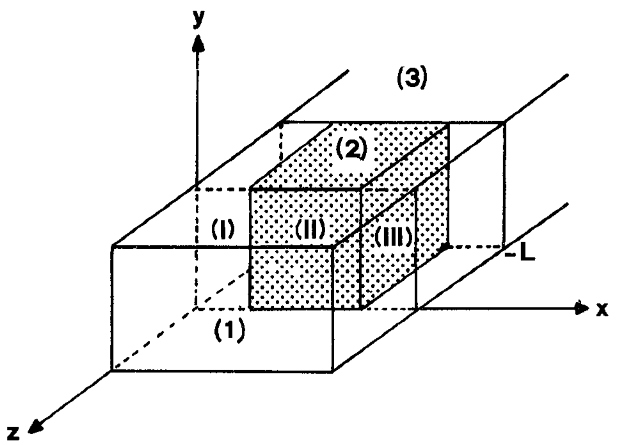

Fig. 2. Rectangular post in a rectangular waveguide.

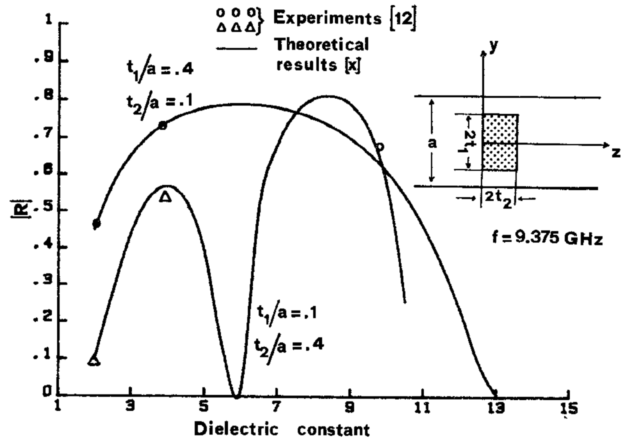

Fig. 3. Theoretical and experiment results of the magnitude $|R|$ of the post as a function of the dielectric constant. The cross-sectional sizes are shown in the figure.

regions:

1) input region;
2) interaction region with three (I, II, III) subregions;
3) output region.

The field is expanded in TE_{m0} modes in the first and third regions and in the corresponding modes of a strip loaded waveguide in the second region.

The incoming electric field is

$$E_y^i(x,z) = \sin(\gamma_1 x) e^{ju_1 z} \quad (4)$$

where

$$\gamma_1 = \frac{\pi}{a} \qquad u_1 = (k_0^2 - \gamma_1^2)^{1/2} \qquad k_0 = \frac{2\pi}{\lambda_0}.$$

In the first and third regions the field is expressed as follows:

Input Region:

$$E_y^1(x,z) = \sum_1^\infty D_m \sin(\gamma_m x) e^{-ju_m z}. \quad (5)$$

Output Region:

$$E_y^3(x,z) = \sum_1^\infty E_m \sin(\gamma_m x) e^{ju_m z}. \quad (6)$$

Here $m = 1,2,3,\cdots, \gamma_m = m\pi/a$, $u_m = (k_0^2 - \gamma_m^2)^{1/2}$, and D_m and E_m are arbitrary complex coefficients. As we can see in the present case, we have odd and even eigenvalues. This did not happen in previously reported work [4], where we had only odd eigenvalues. The existence of both types of eigenvalues comes from the field which is produced in the interaction region. The electric field for a given frequency in the interaction region can have odd or/and even symmetry.

In the interaction region, the field will be expressed separately in each of the three subregions. In subregions I and III, for symmetry reasons, the field must have a symmetric expression. So, in our procedure we can use the expressions of the fields in subregion I. This is

$$E_y^{\rm I} = \sum_1^\infty \sin(h_1^i x)(A_i^0 e^{-jk_1 z} + B_i^0 e^{jk_1 z}). \quad (7)$$

In the subregion II the field will be

$$E_y^{\rm II} = \sum_1^\infty [A_i \cos(h_2^i x) + B_i \sin(h_2^i x)] \cdot (A_i^0 e^{-jk_1 z} + B_i^0 e^{jk_1 z}) \quad (8)$$

where k_1^i is the propagation constant of the ith mode of the strip-loaded waveguide, and A_i^0 and B_i^0 are the complex coefficients of the transmitted and the reflected waves in the interaction region. A_i and B_i are complex expansion coefficients which are selected to satisfy the continuity condition of the fields at $x = d$ and $x = d + t$:

$$h_1^i = \left[k_{\rm I}^2 - (k_1^i)^2\right]^{1/2}$$
$$h_2^i = \left[k_{\rm II}^2 - (k_1^i)^2\right]^{1/2}. \quad (9)$$

For the evanescent modes in the interaction region the term $(A_i^0 e^{-jk_1 z} + B_i^0 e^{jk_1 z})$ is replaced by $C_i^0 e^{-jk_1 z}$. This is because k_1^i becomes imaginary and the field is attenuated along z.

Using Maxwell's equations we can find the corresponding components of the magnetic fields in the input region, the interaction region, and the outpout region.

The coefficients A_i and B_i have already been chosen to satisfy the continuity conditions at $x = d$ and $x = d + t$. The other coefficients, A_i^0, B_i^0, C_i^0, E_m, and D_m, must be chosen to satisfy the continuity conditions at $z = 0$ and $z = -L$. From this we obtain four complex linear equations. By taking the inner product of each equation by $\sin(\gamma_m x)$, we get a system of linear equations with the unknowns A_i^0, B_i^0, C_i^0, E_m and D_m. For a finite number of terms in the summations of the field expressions the system has a finite number of unknowns. In (10) a general expression of the system is given:

$$
\left[\begin{array}{c|c|c|c|c|c}
 & N & N & M & M & K \\
\hline
N & \begin{matrix} a/2 & & 0 \\ & \ddots & \\ 0 & & a/2 \end{matrix} & 0 & S_j^i & S_j^i & S_{i+N}^j \\
\hline
N & 0 & \begin{matrix} -(a/2)e^{-ju_14} & & \\ & & 0 \\ & \ddots & \\ 0 & & \\ & -(a/2)e^{-ju_NL} & \end{matrix} & S_j^i e^{jk_1^iL} & S_j^i e^{-jk_1^iL} & S_{i+N}^j e^{-jk_1^{(i+N)}L} \\
\hline
N & \begin{matrix} ju_1a/2 & & \\ & & 0 \\ & \ddots & \\ 0 & & \\ & & ju_Na/2 \end{matrix} & 0 & -jk_1^iS_j^i & jk_1^iS_j^i & -jk_1^{i+N}\cdot S_{i+N}^j \\
\hline
N & 0 & \begin{matrix} -ju_1(a/2)e^{-ju_1L} & & \\ & & 0 \\ & \ddots & \\ 0 & & \\ & -ju_N(a/2)e^{ju_NL} & \end{matrix} & -jk_1^iS_j^ie^{jk_1^iL} & jk_1^iS_j^ie^{-jk_1L} & \begin{matrix}(-jk_1^{i+N}) \\ S_{i+N}^j e^{-jk_1^{(i+N)}L}\end{matrix} \\
\hline
 & & & 2M+K=2N & &
\end{array}\right]
\left[\begin{array}{c} \\ \hline D_1 \\ \cdot \\ \cdot \\ D_N \\ \hline E_1 \\ \cdot \\ \cdot \\ E_N \\ \hline A_0^1 \\ \cdot \\ \cdot \\ A_0^M \\ \hline B_0^1 \\ \cdot \\ \cdot \\ B_0^M \\ \hline C_0^1 \\ \cdot \\ \cdot \\ C_0^K \end{array}\right]
=
\left[\begin{array}{c} \\ \hline \frac{a}{2} \\ 0 \\ \cdot \\ \cdot \\ \hline ju_1\frac{a}{2} \\ 0 \\ \cdot \\ \cdot \\ \hline 0 \\ \cdot \\ \cdot \\ \hline 0 \\ \cdot \\ \cdot \\ \hline 0 \\ \cdot \\ \cdot \end{array}\right]
\tag{10}
$$

Solving the system for the unknowns, we get D_1 and E_1, which give the reflection and the transmission coefficients. If the number of terms is increased and a negligible change in the reflection and transmission coefficients results, it is assumed that the process is convergent and that a sufficient number of terms have been selected for a given order of accuracy.

The elements S_i^j of (10) have three terms:

$$S_i^j = S_{i,1}^j + S_{i,2}^j + S_{i,3}^j \tag{11}$$

$$S_{i,1}^j = \frac{\sin\left[\left(h_1^i - \gamma_j\right)d\right]}{2\left(h_1^i - \gamma_j\right)} - \frac{\sin\left[\left(h_1^i + \gamma_j\right)d\right]}{2\left(h_1^i + \gamma_i\right)} \tag{12}$$

$$
\begin{aligned}
S_{i,2}^j = A_i\Bigg[&\frac{1}{\left(\gamma_j - h_2^i\right)}\sin\left(\frac{\left(\gamma_j - h_2^i\right)a}{2}\right)\sin\left(\left(\gamma_j - h_2^i\right)\left(\frac{a}{2} - d\right)\right) + \frac{1}{\left(\gamma_j + h_2^i\right)}\sin\left(\frac{\left(\gamma_j + h_2^i\right)a}{2}\right)\sin\left(\left(\gamma_j + h_2^i\right)\left(\frac{a}{2} - d\right)\right)\Bigg] \\
+ B_i\Bigg[&\frac{1}{\left(h_2^i - \gamma_j\right)}\cos\left(\frac{\left(h_2^i - \gamma_j\right)a}{2}\right)\sin\left(\left(h_2^i - \gamma_j\right)\left(\frac{a}{2} - d\right)\right) + \frac{1}{\left(h_2^i + \gamma_j\right)}\cos\left(\frac{\left(h_2^i + \gamma_j\right)a}{2}\right)\sin\left(\left(h_2^i + \gamma_j\right)\left(\frac{a}{2} - d\right)\right)\Bigg]
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
S_{i,3}^j = \sin\left(h_1^ia\right)&\left[-\frac{\cos\left(\left(\gamma_j - h_1^i\right)a\right)}{2\left(\gamma_j - h_1^i\right)} - \frac{\cos\left(\left(\gamma_j + h_1^i\right)a\right)}{2\left(\gamma_j + h_1^i\right)} + \frac{\cos\left(\left(\gamma_j - h_1^i\right)(a-d)\right)}{2\left(\gamma_j - h_1^i\right)} + \frac{\cos\left(\left(\gamma_j + h_1^i\right)(a-d)\right)}{2\left(\gamma_j + h_1^i\right)}\right] \\
-\cos\left(h_1^ia\right)&\left[\frac{\sin\left(\left(h_1^i - \gamma_j\right)a\right)}{2\left(h_1^i - \gamma_j\right)} - \frac{\sin\left(\left(h_1^i + \gamma_j\right)a\right)}{2\left(\gamma_j + h_1^i\right)} - \frac{\sin\left(\left(h_1^i - \gamma_j\right)(a-d)\right)}{2\left(h_1^i - \gamma_j\right)} + \frac{\sin\left(\left(h_1^i + \gamma_j\right)(a-d)\right)}{2\left(h_1^i + \gamma_j\right)}\right].
\end{aligned}
\tag{14}
$$

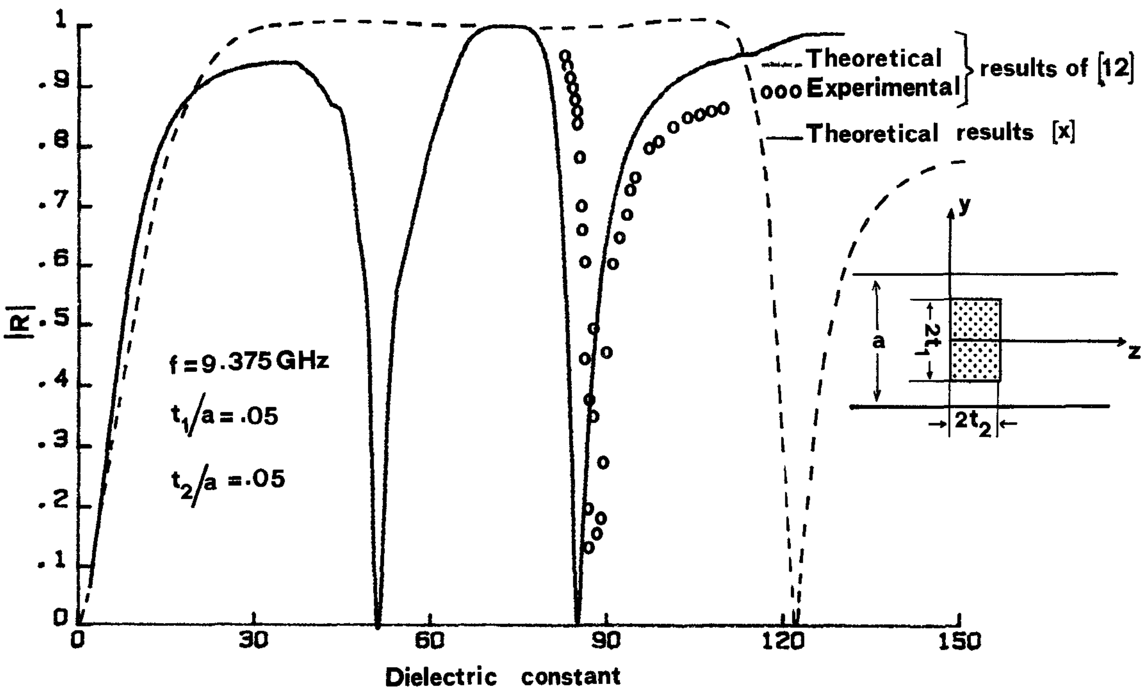

Fig. 4. Theoretical and experimental results of $|R|$ versus the dielectric constant for the rectangular post shown in the figure.

If we suppose that the frequency produces only one transmission mode (mode with real propagation constant), we can find the reflection and transmission coefficients by eliminating all the other coefficients of the evanescent modes:

$$R = D_1 = 2\frac{1+\dfrac{k_1^1}{u_1} - e^{-2jk_1L}\left(1-\dfrac{k_1^1}{u_1}\right)}{\left(1+\dfrac{k_1^1}{u_1}\right)^2 - e^{-2jk_1L}\left(1-\dfrac{k_1^1}{u_1}\right)^2} - 1 \quad (15)$$

$$T = E_1 = \frac{4e^{j(u_1-k_1)L}\left(2\dfrac{k_1^1}{u_1}\right)}{\left(1+\dfrac{k_1^1}{u_1}\right)^2 - e^{-2jk_1L}\left(1-\dfrac{k_1^1}{u_1}\right)^2}. \quad (16)$$

In the general case we can solve (10) and find the reflection and the transmission coefficient at the discontinuity.

Looking at the form of our solution and the form of that given by Yoshikado and Taniguchi [13], we can easily see that their expression (5) for the field in the rectangular post region fits only for one transmitting mode. As the thickness of the dielectric post increases we have a remarkable increase in the number of transmission modes. For example if $\epsilon_1 = 1$ and $\epsilon_2 = 7.5$ for a frequency of 9 GHz, the number of transmission modes varies between one and four as the thickness varies from $t/a = 0.1$ to 0.4. So a difference between the numerical and the experimental results in [13] must be naturally expected.

It must be pointed out that, in general, the number of transmission modes increases with the frequency, the dielectric constants ϵ_1 and ϵ_2, and the thickness of the larger dielectric constant material.

III. Numerical Results

In this section, computed values of the reflection coefficient for a given post as a function of the dielectric constant are presented. In all cases we checked the term $|R|^2 + |T|^2$ (R the reflection and T the transmission coefficient), which for a lossless post must be equal to unity. Using a system with all the transmission modes M, and $K(K < M)$ evanescent modes, we solve system (10). K starts from zero and increases until $1 - |R|^2 - |T|^2 < 10^{-11}$. From the numerical results we found that it must be equal to zero or at maximum to 1. The computational time is about 5 s at maximum for a 4381 IBM computer. In Fig. 3 $|R|$ is shown for two different cross sections. In the same figure are shown some experimental results given in [13].

A similar case is also shown in Fig. 4, where our results are compared with those given in [13]. From the comparison we can see that our results are in good agreement with experiments. Especially in Fig. 4 our method makes the prediction of resonance very close to the experiments of [13]. At the same time we can see that we have another resonance for $\epsilon_r \sim 45$. It must be pointed out that a post with $\epsilon_r \sim 80$–90 has four transmission modes. These are important in predicting the exact value of $|R|$. Another case is shown in Fig. 5, where $|R|$ is compared with the values given by Uher *et al.* [15]. Looking at the three

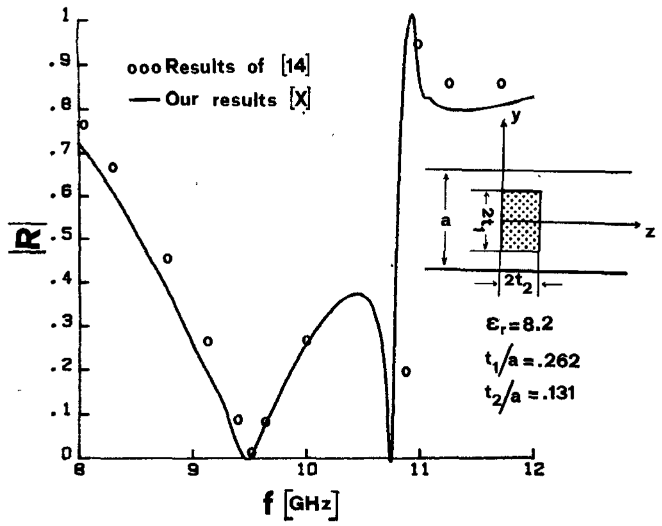

Fig. 5. Comparison of numerical results of $|R|$ versus the frequency for a post by two different methods.

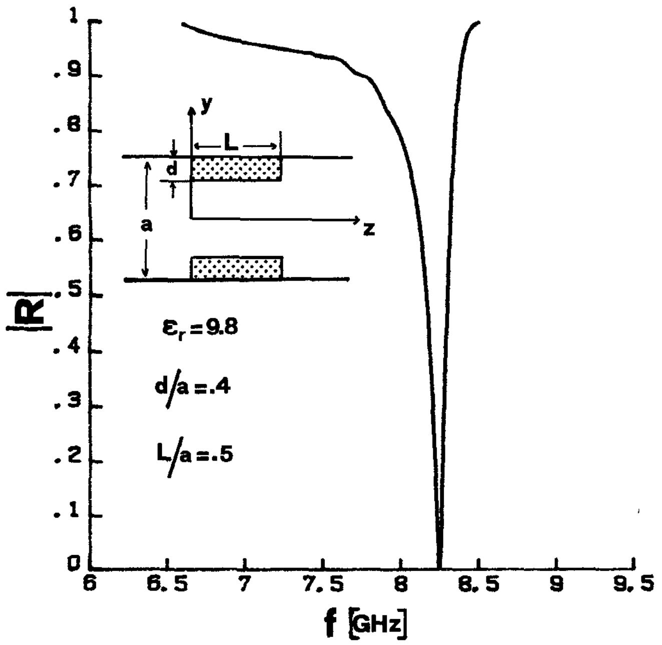

Fig. 6. $|R|$ of an air-filled rectangular post with two dielectric strips versus frequency.

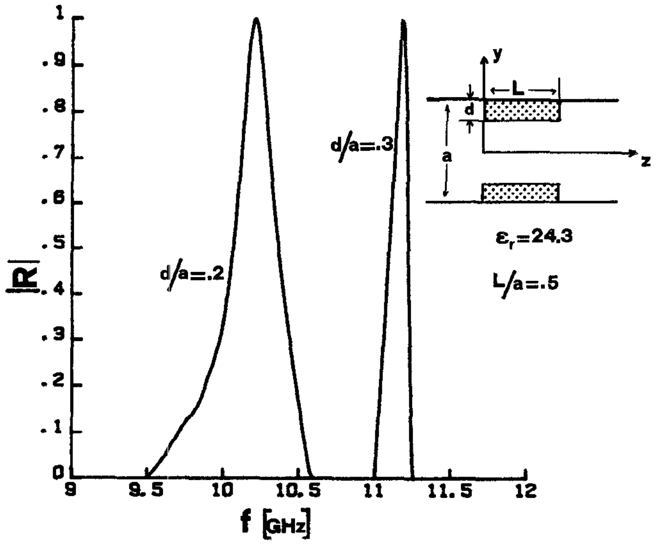

Fig. 7. Band-stop filter response of an air-filled rectangular post for two different strip thicknesses.

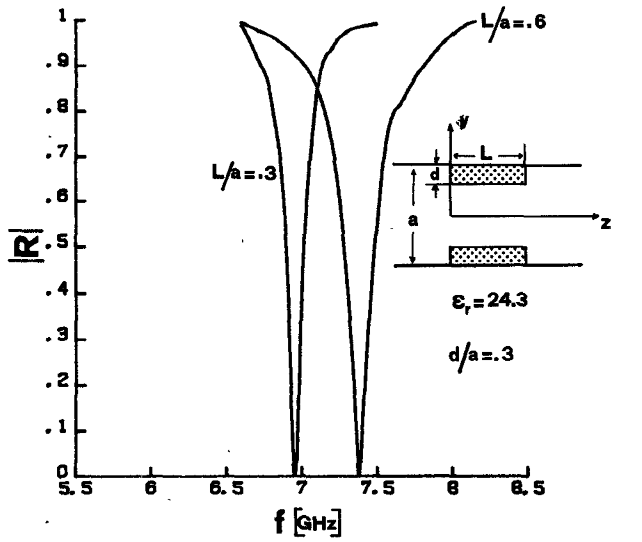

Fig. 8. Band-pass filter response of an air-filled rectangular post for two different strip lengths.

figures, we can conclude that our method gives more accurate results than other methods and is in agreement with the experiments.

Some new cases can be seen for the design of band-pass and band-stop filters with different configurations. Choosing an air-filled rectangular post with two dielectric strips on the two vertical conductors of the waveguide, we have another interesting case. For the geometry given in Fig. 6 we can see that we have a resonance at 8.25 GHz. The above case suggests a band-pass filter with sintered Al_2O_3 ceramic strips.

For a band-stop filter, we choose the geometry given in Fig. 7. Different thicknesses of the strips give different resonant frequencies. By changing the strip length one can see that the filters become band-pass (see Fig. 8).

The above three cases make it evident that filters with simple geometries and readily available materials can be made.

IV. Conclusion

A numerical method has been given to analyze rectangular dielectric posts in the middle of a rectangular waveguide. From the method it has been found that it is important to use all the transmitting modes in the interaction region. A comparison with experimental results has shown the validity of our procedure. Also, some useful examples for the design of band-pass and band-stop filters have been presented.

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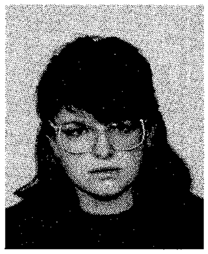

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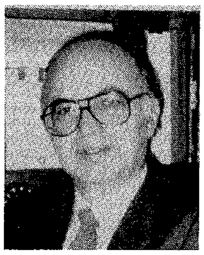

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